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ABSTRACT

This paper outlines three key characteristics of mathematics and how they play out in mathematics education and mathematics education research: (1) the two faces of mathematics; (2) the developmental nature of mathematics; and (3) mathematics as cultural construction. (KHR)

Complexity of Mathematics in the Real World

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Perception of the relationships among mathematics, mathematics education, and mathematics education research used to be simple.

Mathematics was seen as a relatively well-defined, hierarchically structured, body of knowledge. Mathematics education meant transmitting this body of knowledge to each student up to an appropriate level in the hierarchy. Psychological research was expected to provide general theories of cognitive development and learning, with the assumption that these theories could be applied to the learning of mathematics as a domain and the improvement of (mathematics) education through generating hypotheses testable via standard experimental designs. Many mathematicians and psychologists taking a more or less informed interest in mathematics education feel comfortable with this simplicity (for example, in the context of the Californian Math Wars, see the analysis of the unholy alliance between psychologists and mathematicians by Jacob and Akers (1999)).

However, the situation has become more complicated.

First, mathematics continues to grow fast and computers have changed both its content and its methods. Consequently, questions of selection and arrangement arise – what parts of mathematics should be chosen and how should they be reorganized for education? Typically, curricula are largely the result of tradition and inertia and, insofar as growth occurs, it is mainly through accretion without radical restructuring. There is very little by way of principled design – consider the limited adaptation to the new representational systems afforded by computers, for example.

Second, the first wave of the cognitive revolution generated disequilibrium when it became clear that there was "de-emphasis on affect, context, culture and history" (Gardner, 1985, p. 41). The outcome was the "second wave" (De Corte, Greer, & Verschaffel, 1996, p. 497) which mathematics education research both contributed to, and was influenced by, in major ways. Methodologies became interpretative rather than scientific, with results that are liberating or anarchical, depending on your point of view. The work of some researchers now exemplifies Engestrom's proposed methodology for activity theory that puts it to "the acid test of practical validity and relevance in interventions that aim at the construction of new models of activity jointly with the local participants" (Engestrom, 1999, p. 35).

Inevitably, mathematics education researchers' views of mathematics have been complicated by their immersion in activity systems, including exposure to the culture of the classroom, the nature of schooling, and the politics of mathematics education. Mathematics, mathematics education, mathematics education research are all situated in sociohistory, culture, and politics.

To illustrate the foregoing comments, I offer sketchy outlines of three key characteristics of mathematics (revealing my own biases, naturally) and how they play out in mathematics education and mathematics education research.

The Two Faces of Mathematics

> On the one hand, mathematics is rooted in the perception and description of the ordering of events in time and the arrangement of objects in space, and so on ("common sense -- only better organized", as Freudenthal (1991, p. 9) put it), and in the solution of practical problems. On the other hand, out of this activity emerge symbolically represented structures that can become objects of reflection and elaboration, independently of their real-world roots. In the process, common sense is soon transcended, yet, time and again, the results of such elaborations have proved (often after a considerable lag in time) useful in theoretical descriptions of real-world phenomena and solution of real-world problems. (De Corte, Greer, & Verschaffel, 1996, p. 500).

The link between the two faces of mathematics is the activity of modeling. Typically, the modeling of a real-world situation leads to a range of solutions that need to be judged in terms of human criteria such as utility, purpose, and complexity. Introducing pupils early to this perspective may be considered part of the process of enculturation into the practices of mathematicians, yet until relatively recently, it has not received much attention (Niss, 2001; Verschaffel, 2002).

The Developmental Nature of Mathematics

"Mathematics grows ... by its self-organizing momentum" (Freudenthal, 1991, p. 15). In the course of the sociohistorical construction of mathematics, several developmental mechanisms may be identified:

(a) The disequilibrium that comes from lack of closure. The obvious example is the extension of the concept of number from its origins in natural numbers. (It seems to me that there is a clear parallel with Piagetian theory but I am not aware of anyone who has explored this idea in depth).

(b) Metaphorical extension, which has been elaborated in the recent book by Lakoff and Nunez(2000) (and see Edwards). Why are all those different things all called "numbers"? (Poincare defined mathematics as the art of giving the same name to different things).

(c) Variations on the theme of reification (e.g. Sfard, 1991, and see Vinner).

(d) Mediation by cognitive tools, as illuminated by the Vygotskian tradition – language (see Sfard), symbols, representational systems (see Goldin, Meissner).

(e) Systematization, including the development of axiom systems. The history of attempts to teach mathematics on this basis is well known.

It has been pointed out that a major reason for the difficulty of mathematics education is that children are expected to master in a few years concepts that took humankind millennia to develop. All of the above developmental processes have ramifications at the ontological level. In particular, analyses of developmental obstacles represent one broad focus for the continuing relevance and usefulness of cognitive analyses (Greer, 1996).

Mathematics as Cultural Construction

"Mathematics as a human activity" has become a principle cutting across developments in mathematics education, new directions in the philosophy of mathematics education (e.g. Hersh), and influences on mathematics education from critical pedagogy, ethnomathematics, feminist critiques, historical perspectives, and so on.

For balance, it should be remembered that the proof of Fermat's last theorem, and the pages of complex formulae that Ramanujan sent to Hardy also represent human activity and require an account of the coherence and continuity of cognitive processes within an individual brain over an extended period of time however mediated by social environments (Greer, 1996).

Mathematics as a Form of Communication

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Many different answers have been offered to the question *What is mathematics?* throughout history, but the definition given by Henri Poincare is the one which I find particularly useful. According to the French mathematician, *mathematics is the science of calling different things the same name*. This deceptively simple statement, if interpreted in a way not necessarily intended by Poincare himself, can be seen as a forerunner of the *communicational* vision of mathematics. In what follows, I outline this special